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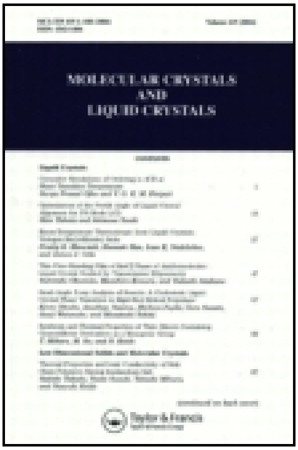

Molecular Crystals and Liquid Crystals Science and Technology. Section A. Molecular Crystals and Liquid Crystals

Publication details, including instructions for authors and subscription information:
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OPTICAL SPATIAL SOLITONS IN LIQUID CRYSTALS

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Version of record first published: 24 Sep 2006

To cite this article: S. K. Srivatsa & G. S. Ranganath (2001): OPTICAL SPATIAL SOLITONS IN LIQUID CRYSTALS, Molecular Crystals and Liquid Crystals Science and Technology. Section A. Molecular Crystals and Liquid Crystals, 366:1, 337-358

To link to this article: http://dx.doi.org/10.1080/10587250108023977

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Mol. Cryst. and Liq. Cryst., 2001, Vol. 366, pp. 337-358
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Optical Spatial Solitons in Liquid Crystals

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We have worked out the structure of optical spatial solitons in liquid crystals. The optical nonlinearity in these systems can be due to (i) Director reorientation, (ii) Suppression of the director fluctuations, (iii) Change of tilt angle in smectic liquid crystals, and (iv) Thermal indexing. When we considered the combined effect of laser suppression of the director fluctuation and thermal indexing we not only get the usual soliton solutions but also find a kink soliton solution. When both the third and fourth processes are operating we find a soliton solution with the nonlinearity which can continuously change from one type to the other. We have worked out the critical laser power required for the formation of solitons in different cases. We have also addressed ourselves to nonlinear optical effects on light propagation in liquid crystals. In the case of beams with large width for which diffraction can be ignored we find interesting modulation of the beam width in a nematic due to the second and fourth processes.

Keywords: Liquid crystals; Optical nonlinearity; Director reorientation; Thermal indexing; Spatial solitons; Beams

INTRODUCTION

Structural solitons in liquid crystals have been investigated extensively by many investigators [1]. Yet the optical solitons in liquid crystals have not received enough attention. These optical solitons could be either spatial or temporal solitons. The spatial solitons in other materials have been extensively investigated in view of their importance in applications [2]. The optical nonlinearities of a medium will have to be considered along with not only the usual diffraction but also dispersion effects. In some media the effects of diffraction and or optical dispersion on the beam are opposite in sense to that due to the nonlinearity. At a certain intensity the nonlinear effect can balance exactly the effect of diffraction or dispersion. This results in beams which travel in the medium without change of spatial intensity profile or pulse shape. If the nonlinearity balances the effects of dispersion, the resulting soliton is termed a *temporal soliton*. On the other hand, if the nonlinearity compensates for the effects

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of diffraction then the resulting soliton is referred to as a *spatial soliton*. In case of spatial solitons it is possible that the nonlinearity could be either positive or negative. If the nonlinear coefficient is positive the resulting soliton will have a central peak with vanishing asymptotes. In literature this is called the *bright* soliton. On the other hand if the nonlinear coefficient is negative then the soliton solution has a central dip in the intensity on a uniform intense background. Such solitons are called *dark* solitons[3]. It should be remarked here that the term *soliton* refers to special solutions which preserve their shape after a pairwise collision. They are usually described by a system of differential equations which are completely *integrable*. It is known that only the Kerr nonlinearity leads to an integrable differential equation. However, all of the nonlinearities described so far do not lead to integrable differential equations and they possess only solitary wave solutions. Unfortunately solitary waves are also sometimes referred to in the literature as solitons. In our studies we get only solitary waves.

Lam et. al.,[5, 6] were probably the first to draw attention to the optical solitons in liquid crystals. They have suggested that the laser induced director reorientation as the nonlinear process involved in the formation of temporal and spatial solitons. Rodriguez et. al., [7, 8] have also studied this problem in the waveguide geometry. These authors, however, did not take into account the inevitable director relaxation process in liquid crystals. Director relaxation is a very slow process and has time scales of the order of a few milliseconds to seconds. However, the optical pulse propagates very fast, in fact, with the velocity of light in the medium. Since the field is non-zero only for a very short duration and the medium does not respond to the field within that time interval there are conceptual difficulties in accepting the suggested solutions for the temporal solitons. In contrast, the process of director reorientation does not lead to such conceptual difficulties in the case of spatial solitons. As a matter of fact, Warenghem et. al., [9] have provided experimental evidence for the existence of spatial solitons. But, director reorientation is not the only nonlinear optical process operating in liquid crystals. It has been shown recently by us [10] that laser suppression of director fluctuations and the laser induced tilt angle in smectic liquid crystals also leads to optical nonlinearities. The first process, though not as strong as the process of director reorientation, is still much stronger than the classical Kerr process. The nonlinearity due to the second process is comparable to that due to the director reorientation. It is well established that laser absorption leads to heating of the sample resulting in a change of refractive index. This process is also associated with a considerable optical nonlinearity and in literature is termed as 'thermal indexing'[11]. The relevant nonlinear coefficients can be both positive

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and negative depending on the geometry and the polarisation of the laser field. In some situations we can neglect the reduction of the intensity due to absorption. This being due to weak absorption or small sample thickness. This limit is known in the literature as the $thin - film$ approximation[4].

In this paper we consider only spatial solitons in liquid crystals. We have worked out [10], the structure of the spatial solitons in the presence of the new nonlinearities referred to above. In the presence of thermal indexing alone, Bertolotti, et. al., [12] have found out numerically that stable soliton like propagation are possible for small distances. They have used the glass as a nonlinear medium. Here, the rigorous solution requires one to solve for both the temperature profile and the field amplitude simultaneously. The temperature profile is obtained from the thermal diffusion equation and thus this process provides a diffusive nonlinearity. It has been stated already that in liquid crystals the nonlinearity could be positive or negative. When this nonlinearity balances exactly the diffraction effect we get depending upon the sign of the nonlinearity either a 'bright soliton or a 'dark' soliton [3]. We find that the combined effect of nonlinearities due to suppression of the director fluctuations and thermal indexing leads to a kink soliton solution and also the usual solitary wave solution with the bell shaped profile. It must be mentioned here that all kinks are solitons, but not all solitons need be kinks. We also find soliton solutions in smectics with thermal indexing operating along with the process of laser induced change in tilt angle.

We next work out the critical laser power at which the effects of diffraction is exactly balanced by that of nonlinearity leading to solitons. We find that in case of laser suppression of director fluctuation alone, the critical power is proportional to the beam cross-section while in cases with both suppression of director fluctuations and thermal indexing there are two critical powers for which the solitons can occur.

Studies on spatial and temporal transformations of optical beams have also become important in recent times. The changes in the beam properties inside the medium provide information about the material properties of passive and active media. Such changes are due to the presence of various nonlinear processes in the media. The spatial variations include the changes in the intensity profile and beam width and temporal variations are inevitable in case of non-stationary processes[4].

We consider the consequences of nonlinearities when a very wide beam is incident on the sample. In such a limit the effects of diffraction can be neglected. We consider a nematic liquid crystal in a laser beam, linearly polarised with the electric vector parallel to the director. Two nonlinear processes, i.e., the suppression of director fluctuations and thermal indexing are assumed to be operating. We find that, in

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this case there are periodic spatial oscillations in the beam width as it propagates inside the medium. If thermal indexing dominates, the beam diverges and when the other process dominates the beam converges. For unpolarised light, the same process leads to a far field pattern in which the central spot has its electric vector perpendicular to the director and is surrounded by an orthogonally polarised outer region. We next take up a nematic with a hybrid alignment. In this case we find that the beam convergence or divergence during initial stage can be exactly balanced by the medium on propagating through the sample during its later stage. Finally, we consider beam propagation in a flexoelectric lattice. It is well known [13] that in the presence of a static electric field we get a splay-bend flexoelectric lattice. The thermal indexing in this case is also periodic and leads to a periodically modulated beam width.

THEORY

It is well established[14, 15] that a magnetic or an electric field applied along the director in a nematic suppresses thermal fluctuations in the director orientation. That a very similar process comes into operation even in a laser field was pointed by us recently [10]. This process leads to nonlinear optical effects. The nonlinear optical coefficient can be considerably large and is positive. The consequent increase in the refactive index for light polarised along the director is given by:

$$\begin{aligned} \delta\mu(I) &= k_B T\sqrt{\frac{\Delta\epsilon^3}{2\pi^3 K^3}}\,\sqrt{I} \\ &= \eta_1\sqrt{I} \end{aligned} \qquad (1)$$

where $\delta\mu$ is the change in refractive index, k_B is the Boltzmann's constant, T is temperature, K is the curvature elastic constant. $\Delta\epsilon = (\epsilon_{\parallel} - \epsilon_{\perp})$ is the dielectric anisotropy. Here $\epsilon_{\parallel}$ and $\epsilon_{\perp}$ are the dielectric constants parallel and perpendicular to the director.

Naturally, the same process affects even the in-plane fluctuations in the c-director of a smectic C liquid crystal. It was also suggested by us [10] that a laser induced change in the tilt order parameter in smectic liquid crystals again leads to large optical nonlinearities which are comparable to the giant optical nonlinearity due to the director reorientation[10, 16, 17]. In smectic A the laser induced tilt angle is possible beyond an intensity threshold and the correction to the refractive index is

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positive. The magnitude of the nonlinear coefficient depends on the direction of propagation and the polarisation of the laser beam.

In addition to the above two processes there can be an additional nonlinear process due to laser absorption by the medium which heats up the material with a consequent change in the refractive index. The refractive index change is related to change in temperature as:

$$\begin{aligned} \delta\mu(I) &= \frac{d\mu}{dT}\left(t^2\chi/\pi^2\kappa\right) I. \\ &= \eta_2 I \end{aligned} \tag{2}$$

Here t is the sample thickness, χ is the optical absorption coefficient, and κ is the thermal conductivity of the medium. The raise in temperature could be as large as 10 K for an absorption coefficient of $0.01cm^{-1}$ [11]. From symmetry considerations it is easy to see that in nematic liquid crystals the ordinary and extraordinary refractive indices should suddenly change to a single refractive index at the nematic-isotropic phase transition. In the nematic phase the extraordinary refractive index decreases and the ordinary refractive index increases with the raise of temperature. This process is referred to in literature as Thermal Indexing. If the laser polarisation is such that the electric vector is parallel to the director the change in refractive index $\delta\mu$ is negative. For an incident beam with intensity peak at the center if the electric vector of the laser beam is parallel to the director this leads to self-divergence as shown in Figure 1 (a). If the electric vector of the laser beam is perpendicular to the director, the change in the refractive index is then positive. This leads to self-focusing as shown in Figure 1 (b). In the second case the director is assumed to be so strongly anchored that the process of director reorientation is absent. Incidentally, when the director is at an angle θ to the electric vector, in the plane of the electric field and the direction of propagation then the change in the refractive index is given by [11]:

$$\frac{d\mu(\theta)}{dT} = \xi(\theta)\frac{\mu_\perp}{\mu(\theta)}\frac{d\mu_\perp}{dT} \tag{3}$$

Here, $\xi(\theta) = \epsilon(\theta)\left[\frac{\epsilon_\parallel - a\epsilon_\perp}{\epsilon_\parallel\epsilon_\perp} - \frac{(a+1)\sin^2(\theta) - a}{\epsilon_\parallel - \Delta\epsilon\sin^2(\theta)}\right]$ with a defined as: $\frac{d\mu_\parallel}{dT} = -a\frac{d\mu_\perp}{dT}$, $a > 0$, $\epsilon_\parallel = \mu_\parallel^2; \epsilon_\perp = \mu_\perp^2$ are the dielectric constants parallel and perpendicular to the director respectively, $\epsilon(\theta)$ is the effective dielectric constant, and $\mu(\theta)$ is the refractive index as seen by the optical beam when the director makes an angle with the electric vector. We note that for $\xi(0) = 1, \frac{d\mu(0)}{dT} = \frac{d\mu_\parallel}{dT}$ and for $\xi(\pi/2) = -a, \frac{d\mu(\pi/2)}{dT} = \frac{d\mu_\perp}{dT}$. Also there exists certain angle θ_o at which $\frac{d\mu(\theta_o)}{dT} = 0$.

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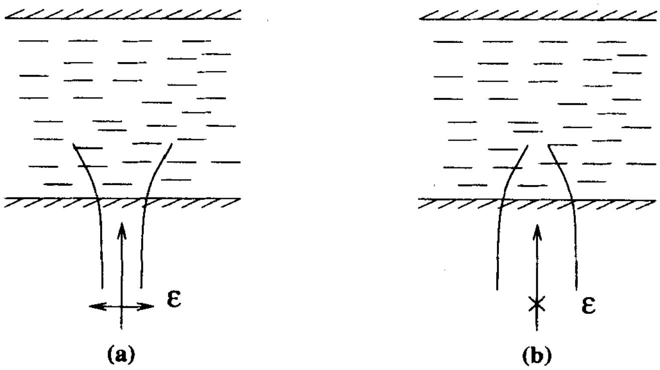

Figure 1: *Effect of thermal indexing on beam width.*
(a) Light propagating in a homogeneously aligned nematic with the electric vector parallel to the director. The beam diverges on propagation.
(b) Light propagating in the homogeneously aligned nematic with the electric vector perpendicular to the director. The beam converges on propagation.

OPTICAL SPATIAL SOLITONS

Optical spatial solitons are optical beams of a certain intensity profile propagating in the nonlinear medium in which the spread of the beam due to diffraction is exactly balanced by either the self-focusing or the self-defocusing of the beam arising from the nonlinearity [18, 19]. We have recently worked out [10] the structure of a few spatial solitons due to the new nonlinear processes of laser suppression of the director fluctuations and change in the tilt angle in smectic liquid crystals[10]. We consider here in addition nonlinearity due to thermal indexing. It should be stressed that when thermal indexing is included, we neglect the reduction in the intensity due to absorption. This is justified, as said earlier, when either the sample thickness is very small or absorption is very weak.

It may be mentioned here that usually the optical soliton formation length in normal Kerr media is about a few kilometers length [6, 20]. But, in liquid crystals, the nonlinear coefficient due to various nonlinear processes are very large compared to the usual Kerr nonlinearity. Interestingly, this leads to very small soliton forma-

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tion length. In fact, it can be of the order of a few micrometer. Further, since in a normal Kerr media large intensities are required to generate spatial solitons [18], the liquid crystals which require much less laser intensity are ideal nonlinear media for the study of spatial solitons.

Nematic Liquid Crystals

(i) *Laser suppression of director fluctuations* :

As said earlier the laser suppresses director fluctuations leading to a sufficiently high optical nonlinearity. The nonlinear refractive index scales as the square root of the incident intensity. Then the Maxwell's equation in the slowly varying envelope approximation becomes identical to the nonlinear Schrödinger equation given by:

$$2ik_o\mu_{\|}\frac{\partial\psi}{\partial\xi}+\frac{\partial^2\psi}{\partial X^2}+2k_o^2\,\mu_{\|}\,\mu_{nl}(I)\psi=0 \tag{4}$$

Here $\psi = \mathcal{E}(X)exp(i\nu\xi)$ is the electric field envelope, $1/\sqrt{\nu}$ directly related to the soliton width. $\xi = k_o * z$ and $X = x * k_o$ are the scaled distances with respect to the wavevector along the propagation direction and the transverse direction respectively, k_o is the wavevector of the laser beam and $\mu_{nl}(I)$ is defined through $\mu(I) = \mu_{\|} + \mu_{nl}(I)$. k_o is the wavevector of the laser beam. The nonlinear refractive index ($\mu_{nl}(I)$) is given by 1 which is positive. The resulting optical soliton is a *bright* soliton described by the following equation [10]:

$$\psi(X,\xi)=\left(\frac{3A}{2B}\right)\frac{1}{\cosh^2(\sqrt{A}\,X/2)}exp(i\;\nu\;\xi) \tag{5}$$

where $A = 2\mu_{\|}k_o\nu$ and $B = 2k_BT\mu_{\|}k_o^2\sqrt{\frac{\epsilon_a^3}{2\pi^3K^3}}$. The profile of this soliton is shown in Figure 2. This soliton should be compared and contrasted with the solution found for a normal Kerr media viz.,

$$\psi(X,\xi)=\left(\frac{2A_o}{B_o}\right)\frac{1}{\cosh(\sqrt{A}\,X)}exp(i\;\nu\;\xi) \tag{6}$$

where $A_o = 2\mu_{\|}k_o\nu$ and $B_o = 2\mu_{\|}k_o^2\,\mu_2$ and μ_2 is the nonlinear coefficient.

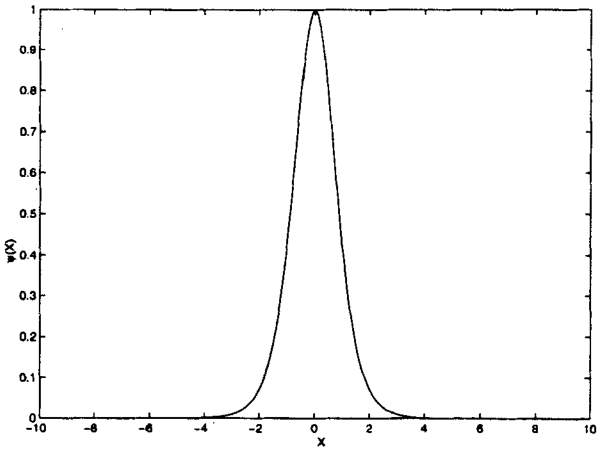

Figure 2: *A typical soliton solution*(5) *of the generalised nonlinear Schrödinger equation (equation* (4)*) resulting due to the process of laser suppression of the director fluctuations alone. X is the transverse coordinate scaled with respect to the wavevector, $X = x * k_o$ and $\psi(X)$ is the amplitude of the laser field.*

(ii) *Thermal indexing* :

Bertolotti, et. al., [12] have studied the optical solitons due to thermal indexing by solving nonlinear Maxwell's equation and the thermal diffusion equation. These propagate only for a short distance before instability sets in. But in liquid crystals due to small sample thickness, low absorption(for optical frequencies) and high nonlinearity we are justified in working in the $thin - film$ approximation. In this limit the Maxwell's equation reduces to the nonlinear Schrödinger equation. Further, we consider the nonlinearity due to this to be local instead of diffusive type of nonlinearity. Thus we can neglect the decrease in intensity due to absorption. Thermal indexing leads to a refractive index change which is either positive or negative. In both the cases the refractive index change is proportional to the laser intensity as in a Kerr media. If the nonlinear coefficient is positive then we get a *bright* soliton described by:

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$$\psi(X,\xi) = \left(\frac{2a}{b}\right) \frac{1}{\cosh(\sqrt{a}\,X)} exp(i\nu\xi) \tag{7}$$

where $a = 2\mu_{\|}k_{o}\nu$ and $b = 2\mu_{\|}k_{o}^{2}\mu_{nl}(I)$. Here $\mu_{nl}(I) = \frac{d\mu}{dT}\delta T(I)$. If the thermal indexing leads to a negative nonlinear coefficient we get a *dark* soliton solution given by:

$$\psi(X,\xi) = \psi_{o}\left[iA + \sqrt{1-A^{2}}\tanh(X)\right] exp(i\psi_{o}^{2}\xi) \tag{8}$$

Here A is a parameter depending on the nonlinear coefficient, ψ_o is a measure of the amplitude of the intense background and the profile is shown in Figure 3.

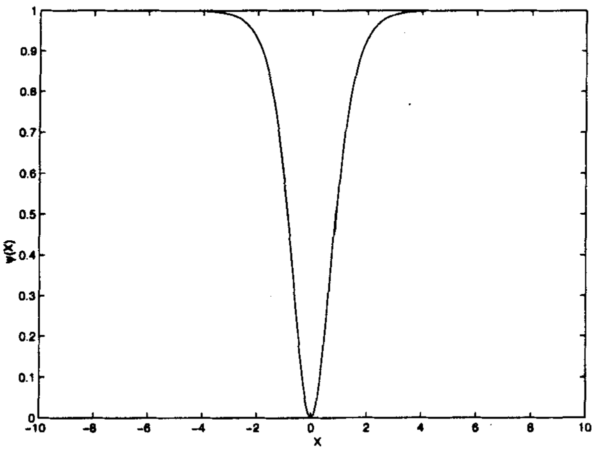

Figure 3: *A typical dark soliton solution*(8) *of the nonlinear Schrödinger equation resulting due to 'thermal indexing' alone. X is the transverse coordinate scaled with respect to the wavevector,* $X = x * k_o$ *and* $\psi(X)$ *is the amplitude of the laser field.*

(iii) *Thermal indexing and the laser suppression of director fluctuations* :
In this case the electric vector of the laser beam is parallel to the director. The thermal nonlinearity here is proportional to $\frac{d\mu_{\|}}{dT}$ and is negative. The corresponding

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nonlinear Maxwell's equation can be written as:

$$i\frac{\partial \psi}{\partial \xi} + \frac{\partial^2 \psi}{\partial X^2} + |\psi|\psi - \gamma|\psi|^2\psi = 0 \tag{9}$$

which is again a nonlinear Schrödinger equation. Here γ is the ratio of the nonlinear coefficients η_1 and η_2 due to the two processes.

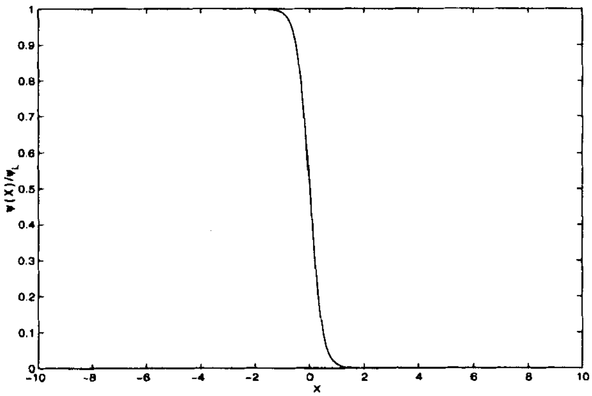

Figure 4: *A kink soliton solution of the nonlinear Schrödinger equation (equation (11)). The parameters in the equation are as follows:* $\psi_L = 60.945$, $\nu = 20.3$, $\zeta = 4.5$ *and* $\gamma = 0.01$. X *is the transverse coordinate scaled with respect to the wavevector,* $X = x * k_o$ *and* $\psi(X)$ *is the amplitude of the laser field.*

The soliton solution for equation (9) with the boundary conditions $|\psi(\pm\infty)| \to 0$ is given by:

$$\psi(X,\xi) = \frac{1}{\left[\frac{1}{3\nu} + exp(-\sqrt{\nu}\,X) - \left(\frac{\gamma}{2\nu} - \frac{1}{(3\nu)^2}\right) exp(\sqrt{\nu}\,X)\right]}\, exp(i\,\nu\,\xi) \tag{10}$$

The solution here has vanishing asymptotes. On the other hand, the kink solution for the same equation i.e., (9), is given by [21]:

$$\psi(X,\xi) = \psi_L \left(\frac{1}{1 + exp(\zeta X)}\right) exp(i\,\nu\,\xi) \tag{11}$$

where, $\psi_L = \frac{2}{3\gamma}; \nu = \frac{2}{9\gamma}; \zeta = \pm\sqrt{2/\gamma}/3$.
The kink profile is shown in the Figure 4. It should be mentioned here that generally kink optical solitons have a phase difference at the two limits i.e., $X = \pm\infty$) which exist only in the case of *dark solitons*. The present kink soliton by contrast does not possess such a phase difference. It should be mentioned here that generally kink optical solitons have a phase difference in the electric field at the two limits i.e., $[\phi(\infty) - \phi(-\infty)]$ and these appear only in the case of *dark solitons*. The present kink soliton by contrast does not possess such a phase difference.

Smectic Liquid Crystals

We now consider solitons in smectic liquid crystals. Here in some geometry both the process of laser suppression of the c-director fluctuations and laser induced change in the tilt angle exist while in some other geometries only the second process operates. We consider two geometries. In the first the light propagates in a smectic A perpendicular to the layers with the electric vector of the laser beam parallel to the layers. In the second geometry light propagates in a smectic C parallel to the layers polarised perpendicular to the layers and in the plane of the tilt. These two cases are associated with different types of nonlinearities leading to a rich class of solitons. For a self-consistent solution of the Maxwell's wave equation, the tilt angle should be obtained by minimising the total free-energy density. The free-energy density is given by[4]:

$$\begin{aligned}\mathcal{F} &= \alpha\,\theta^2 - \alpha''I\,\theta^2 + \beta\,\theta^4 + \textit{higher order terms}\\ &\quad + \textit{coupling terms}\end{aligned}$$

Here $\alpha = \alpha_o(T - T_{AC})$, with T_{AC}, the smectic A-smectic C transition point and $\beta(>0)$ are phenomenological constants in the free-energy density, $\alpha'' = \Delta\epsilon/16\pi c$, c being the velocity of light. Now if the raise in temperature due to laser absorption is taken into account the free-energy density can be written as:

$$\begin{aligned}\mathcal{F}_{AC} &= \alpha\,\theta^2 + \alpha_o\,\delta T(I)\,\theta^2 - \alpha''I\,\theta^2 + \beta\,\theta^4 + \textit{higher order terms} \\ &\quad + \textit{coupling terms}\end{aligned} \tag{12}$$

where, $\delta T(I) = (t^2\chi/\pi^2\kappa)\ I$, is the raise in temperature due to laser absorption. It

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is easy to show that at a certain threshold intensity given by $I_{th} = \alpha_o(T - T_{AC})/\left[\frac{\Delta\epsilon}{16\pi} - \alpha_o(t/\pi)^2\chi/\kappa\right]$, a smectic A to smectic C transition is induced leading to a non-zero value of θ. The nonlinear coefficient here depends on the geometry i.e., the direction of propagation and the polarisation of the beam. If we make the parameter α in the free-energy density expression (12) zero by going to $T = T_{AC}$ (we can effectively get the same result by an applied static magnetic field), then the nonlinear function $\mu_{nl}(I)$ is the same as Kerr type nonlinearity which possesses a true soliton solution which is to be distinguished from a solitary wave.

Thermal indexing and change of tilt angle in smectics

(i) Case I:

We first consider the case where light propagates parallel to the layer normal with the electric vector parallel to the layers of a smectic A. The system is supposed to be near a smectic A to C transition.

The nonlinear coefficient for thermal indexing in this geometry is positive. A molecular tilt is induced beyond a certain intensity, the threshold for which can be calculated from the free-energy density (12). The nonlinear refractive index $\mu_{nl}(I)$ is given by:

$$\mu_{nl}(I) = \begin{cases} \frac{d\mu_\perp}{dT}\frac{t^2\chi}{\pi^2\kappa} I & , \quad I < I_{th} \\ \frac{(\mu_\parallel^2-\mu_\perp^2)}{2\mu_\parallel^2\beta}\left(\frac{(\mu_\parallel^2-\mu_\perp)^2 I}{8\pi c} - \alpha\right), & I > I_{th} \end{cases}$$

Initially, when the tilt angle is small, the thermal indexing is proportional to $\frac{d\mu(\theta)}{dT}$ which is positive. As the angle increases $\frac{d\mu}{dT}$ becomes zero and changes sign. Thus beyond a threshold intensity the effective nonlinearity decreases with the intensity. The soliton solution in both regimes are Kerr-like solitons given by:

$$\left(\frac{2a_1}{b_1}\right)\frac{1}{\cosh(\sqrt{a_1}\,X)}exp(i\nu\xi) \tag{13}$$

where $a_1 = 2\mu_\perp k_o\nu$ and $b = 2\mu_\perp k_o^2\mu_{nl}(I)$ with $\mu_{nl}(I)$ as given above. The second solution is given by:

$$\left(\frac{2a_2}{b_2}\right)\frac{1}{\cosh(\sqrt{a_2}\,X)}exp(i\nu\xi) \tag{14}$$

where $a_2 = 2\mu_\perp k_o\nu$ and $b_2 = 2\mu_\perp k_o^2\mu_{nl}(I)$ with $\mu_{nl}(I)$ as given above.

(ii) Case II:

Now we deal with light propagation through a smectic C liquid crystal, propagating along the layer with its electric vector parallel to the layer normal. The coefficient of thermal indexing in this case could be positive or negative depending on the tilt angle. Initially it is positive when the angle is large. As the intensity is increases

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the tilt angle changes and $\frac{d\mu(\theta)}{dT}$ becomes negative. Meanwhile the nonlinearity due to change in the tilt angle is always positive. At a certain intensity the tilt angle is zero and this process does not contribute to the nonlinear coefficient thereafter. This intensity is given by:

$$I_{\circ} = \frac{\alpha_{\circ}(T_{AC} - T)}{\left[\frac{\Delta\epsilon}{16\pi} + \alpha_{\circ}(\frac{t}{\pi})^2 \frac{\chi}{\kappa}\right]} \tag{15}$$

At and beyond this intensity the nonlinear coefficient is only due to the thermal indexing and is now proportional to $\frac{d\mu_{\|}}{dT}$ which is negative. The nonlinear refractive index is given by:

$$\mu_{nl}(I) = \begin{cases} \mu_{\|} - \mu(\theta_{\circ}) - \frac{\Delta\epsilon\mu_{\|}|\alpha|}{2\mu_{\perp}^2\beta} + \frac{\Delta\epsilon^2\mu_{\|}}{16\pi c\mu_{\perp}^2\beta} I + \xi(\theta)\frac{\mu_{\perp}}{\mu}\frac{t\mu_{\perp}}{dT}(\frac{t}{\pi})^2\frac{\chi}{\kappa} I, & I < I_{th} \\ \frac{d\mu_{\|}}{dT}, & I > I_{th} \end{cases}$$

where, $\mu(\theta_{\circ})$ is the refractive index without the laser field. We find the solitons[described by the solutions (13) and (14)] is stable on propagation. Further, we get different types of solutions for intensities below and above the threshold even though the nonlinearity is of Kerr-type.

Critical Laser Power For Soliton Formation

In the case of a nonlinear medium with self-focusing nonlinearity, the beam width decreases continuously and at a certain point the beam width becomes comparable to the laser wavelength. At this stage the optical diffraction becomes important. It is known that if the intensity of the laser beam is less than some intensity then the beam converges up to some point and then starts diffracting beyond that point when the process of diffraction takes over. If the power of the incident beam is greater than that of this critical intensity the process of self-focusing dominates over the diffraction. Here the beam continues to converge beyond the diffraction limit destroying the material eventually. These two limits are known to depend on the incident laser power. At a critical power of the laser beam it is found that the convergence due to nonlinearity can be compensated by the diffraction. At this critical power the beam is said to be self-trapped or self-channeled. In recent literature this has come to be known as a soliton.

In order to find the critical power we consider the stability of a plane wave traveling along the z-axis in the nonlinear medium. The steady state nonlinear Maxwell's equation is given by:

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$$ik_{\circ}\frac{\partial E}{\partial z} = -1/2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial y^2}\right)E - \mu_{nl}(I)E \qquad (16)$$

which is a generalised Schrödinger equation. To find the stability of an infinite plane wave we consider the spatial evolution of the perturbations along the direction of propagation. The dispersion relation with the nonlinearity gives for the wavevector:

$$k^2 = \frac{\omega^2\epsilon(\omega)}{c^2} + 2\mu_{nl}(I)$$

$$k = k_{\circ} + \mu_{nl}(I)/k_{\circ}$$

where $k_{\circ} = \omega\epsilon(\omega)/c$, and $\mu_{nl}(I)$ is the nonlinear refractive index. Hence, the amplitude of the unperturbed beam is of the form $E_{\circ}(z) = E_{\circ}\, exp(i\,\mu_{nl}(I)\,z/k_{\circ})$ where $E_{\circ}$ is the amplitude of the incident laser beam. We write the perturbation in the form:

$$\delta E(r) = [\mathcal{A}\, exp(i(\mathbf{q}\cdot\mathbf{r} + k_z z)) + \mathcal{B}^*\, exp(-i(\mathbf{q}\cdot\mathbf{r} + k_z z))]\, exp(i\mu_{nl}(I)z/k_{\circ}) \qquad (17)$$

Here $\mathbf{q}$ is a vector in the $x - y$ plane. Substituting this in the equation (16) and collecting the groups of terms in $exp(\pm i(\mathbf{q}\cdot\mathbf{r} + k_z z))$, we get:

$$\left(\frac{q^2}{2} - \mu_{nl}(I) + k_{\circ}k_z\right)\mathcal{A} - \mu_{nl}(I)\mathcal{B} = 0$$

$$-\mu_{nl}(I)\mathcal{A} + \left(\frac{q^2}{2} - \mu_{nl}(I) - k_{\circ}k_z\right)\mathcal{B} = 0$$

The condition that a nontrivial solution for these equations exist is the determinant of the coefficients vanish. This condition gives: $k_z = \pm\frac{q}{2k_{\circ}}\sqrt{q^2 - 4\mu_{nl}(I)}$. When the nonlinear coefficient is positive, i.e., in a focusing media, if

$$q^2 < 4\mu_{nl}(I) \qquad (18)$$

then k_z becomes imaginary. This gives an exponentially increasing term in the perturbation and the wave is unstable.

In a beam of radius R, the wavevector of perturbation transverse to propagation direction is restricted by $q \geq 1/R$. This sets the upper limit on the q values which is responsible for the focusing instability. Thus the equation (18) gives the critical value of the electric field $E_{\circ}^2 \approx 1/\eta R^2$, if the nonlinearity is of Kerr type where $\mu_{nl}(I) = \eta E_{\circ}^2$. As the power in a laser beam is defined as $E_{\circ}^2 R^2$, the critical power is then given by $P_{cr} \approx 1/\eta$. It is thus independent of the beam width.

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We now consider the same problem in the presence of only the suppression of director fluctuations. The inequality (18) in this case becomes $q^2 < 4\eta_1\sqrt{I}$. The expression for the critical power hence becomes:

$$P_{cr} \approx 1/\left(\eta_1^2 R_o^2\right) \tag{19}$$

where η_1 is the nonlinear coefficient due to laser suppression of the director fluctuations given by (1) and R_o is the radius of the beam. Hence the critical power decreases as the beam size increases.

We next consider the simultaneous presence of suppression of the director fluctuations and the thermal indexing. In this case the condition (18) becomes

$$\left(\eta_1 E_o - \eta_2 E_o^2\right) R^2 \approx 1 \tag{20}$$

This leads to two critical powers and are given by:

$$P_{cr}(1) \approx \left[\frac{\eta_1 R_o - \sqrt{\eta_1 R_o^2 - 4|\eta_2|}}{\eta_2}\right]^2$$
$$P_{cr}(2) \approx \left[\frac{\eta_1 R_o + \sqrt{\eta_1 R_o^2 - 4|\eta_2|}}{\eta_2}\right]^2$$

where η_1 and η_2 are the nonlinear coefficients corresponding to the suppression of the director fluctuations and the thermal indexing respectively. We notice that the critical power exists only if the nonlinear coefficients satisfy the inequality:

$$\eta_1 R_o^2 > 4|\eta_2| \tag{21}$$

Thus we see that irrespective of the strengths of the nonlinear coefficients a width for the beam can be so chosen that the inequality is satisfied. The first(lower) critical power occurs when the effect of beam diffraction exactly balances the effects of suppression of director fluctuations. At this power the thermal indexing does not contribute significantly. The second(higher) critical power occurs when the effect of beam diffraction balances the defocusing effects due to thermal indexing and by the focusing effect of suppression of director fluctuations.

WIDE BEAMS IN ABSORBING NEMATICS

If the beam width is very large compared to the wavelength of the laser then to a very good approximation we can ignore diffraction effects. Yet we cannot ignore absorption effects. In this section we study nematic liquid crystals in such situations.

Uniform Nematic

(i) *Linearly polarised beam*

Let a linearly polarised beam propagate perpendicular to the director with its electric vector parallel to it. In this geometry we get an optical nonlinear effect due to the suppression of the director fluctuations. If the material is also weakly absorbing then the temperature of the system raises. The change $\delta T(I)$ in temperature is proportional to the laser intensity, I. Hence the change $\delta\mu(I)$ in the refractive index is also proportional to I. It is given by (2) and it simulates the familiar Kerr process. It is clear from the discussion presented in section **2** that these two processes compete with each other. In fact the change in the refractive index in the presence of both these nonlinearities can be written as:

$$\delta\mu(I) = \eta_1\sqrt{I} - \eta_2 I \tag{22}$$

The refractive index changes are as shown in Figure 5.

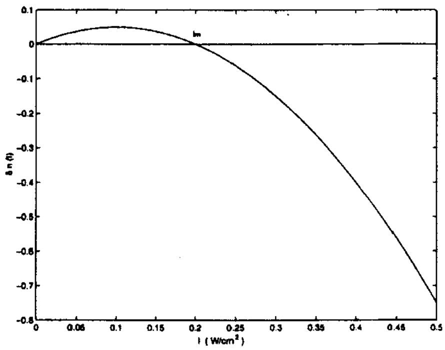

Figure 5: *A typical variation of the refractive index when both the processes, the suppression of the director fluctuations and the thermal indexing are operating in a nematic liquid crystal. I_m is the intensity at which the nonlinearity changes its sign.*

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The change reaches a maximum and becomes zero again at an intensity $I_m = \eta_1^2/\eta_2^2$. Below this intensity the process of laser suppression of director fluctuations dominates and δn is positive while above the intensity I_m the thermal indexing process dominates and δn is negative. The positive nonlinearity leads to self-convergence and the negative nonlinearity to self-divergence of the incident laser beam.

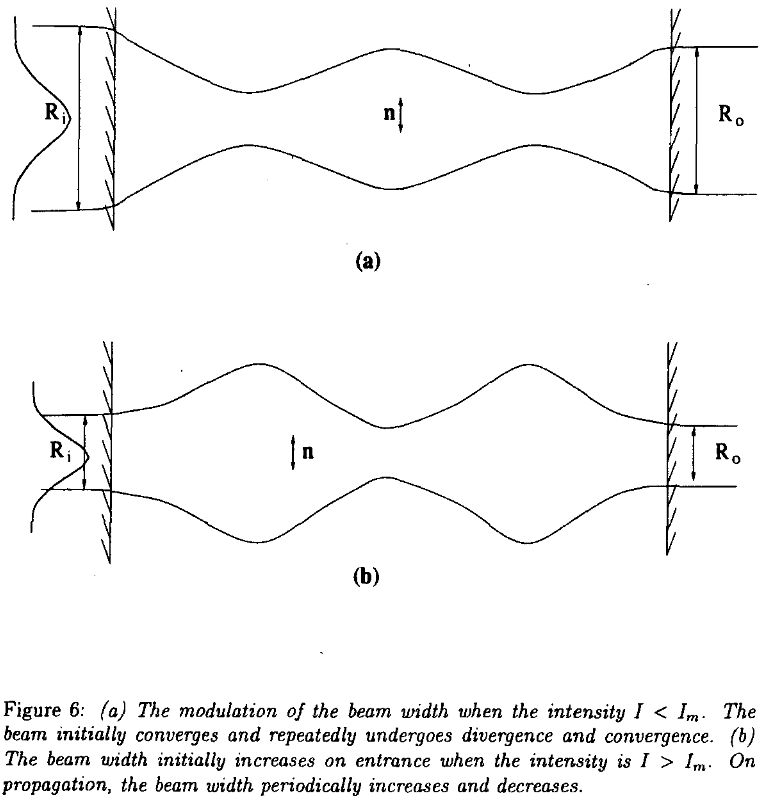

Figure 6: *(a) The modulation of the beam width when the intensity $I < I_m$. The beam initially converges and repeatedly undergoes divergence and convergence. (b) The beam width initially increases on entrance when the intensity is $I > I_m$. On propagation, the beam width periodically increases and decreases.*

Let us chose the intensity I of the incident beam less than I_m. Hence, initially

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the beam converges. This increases the intensity in the beam due to reduction of the cross-section of the beam. As the intensity increases and becomes greater than I_m then the beam starts diverging. This leads to increase in the cross-section of the beam and hence the reduction in the intensity of the beam. At some large width again the beam starts converging. The process repeats indefinitely leading to a periodic modulation of the beam width along the direction of propagation, as depicted in Figure 6 (a). The same is to be expected even in case of $I > I_m$ and the beam width evolution is as depicted in Figure 6 (b).

(ii) *Unpolarised beam*

Laser beams with electric fields along and perpendicular to the director are the eigenwaves in a nematic[23]. As already said the nonlinear coefficient, for the wave propagating in a medium with its electric vector parallel to the director, is negative and for the orthogonal component it is positive. Hence, the component parallel to the director is diverging while the component perpendicular to the director is focusing. Any incident vibration gets resolved into these eigenstates. In the case of unpolarised light the eigenstates will be equally intense. When such a beam traverses through a nematic then due to the optical nonlinearity we get a central bright spot polarised parallel to the director, surrounded by a divergent beam polarised perpendicular to the director.

Hybrid Nematic

Non-uniform director configuration also affects the beam characteristics. To illustrate this situation we consider propagation in a hybrid aligned nematic. In this geometry shown in Figure 7, at one of the bounding glass plates the director is homogeneously aligned i.e., parallel to the surface and at the other bounding glass plate the director is homeotropically anchored i.e., perpendicular to the surface. In the intermediate space the director smoothly goes over from one alignment to the other. If the laser beam is polarised parallel to the director and is incident on the plate with the homogeneous alignment then the beam to start with, will diverge. As it propagates, due to thermal indexing, the negative nonlinear coefficient decreases in magnitude and changes sign at a particular point and becomes positive beyond it. After this stage the beam converges. It is thus possible, by a suitable choice of the thickness, birefringence and absorption coefficients to design a cell where the beam width on emergence is equal to the width it had at the entrance point. On the other hand, in case if the beam is incident on the plate with homeotropic alignment with the electric field in the plane of the director distortion, the beam initially converges

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and eventually diverges on further propagation. Again it is possible to choose the material parameters such that the beam width is the same at the entrance and the exit ends of the sample.

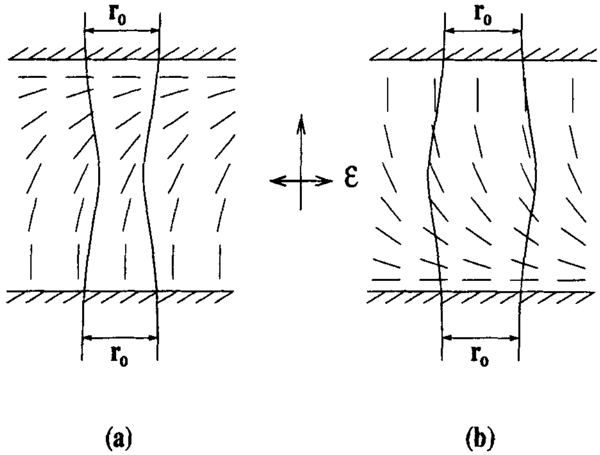

Figure 7: *A hybrid aligned nematic. The beam transformations are shown in the figure. The beam width at entrance is the same at the exit and entrance points. (a) Laser incident on the homeotropically aligned surface. (b) Laser incident on the homogeneously aligned surface.*

Flexoelectric Nematic Lattice

uniform nematic subjected to a static electric field, under certain conditions, exhibits an instability leading to an one dimensional periodic planar splay-bend structure[13]. In nematics with a positive dielectric anisotropy this periodic structure is a result of the competition between the flexoelectric torque and the dielectric torque acting on the director. The wavevector of periodicity is always along the direction perpendicular to the static electric field. A schematic representation of this is shown in Figure 8 (a) with alternate regions of splay and bend deformations.

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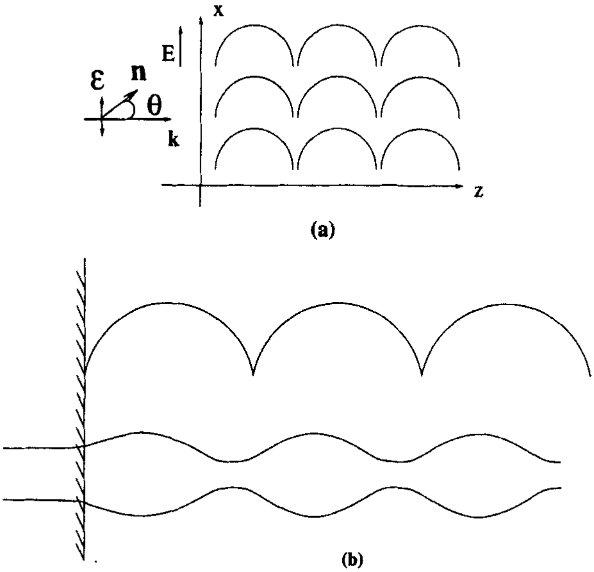

Figure 8: *(a) The flexoelectric lattice in the presence of a static electric field. The periodicity is along the z direction perpendicular to the electric field(x axis). The laser beam is propagating along the periodicity axis. (b) The periodic variation of the beam width due to inhomogeneous periodic nonlinearity induced by thermal indexing.*

Let a linearly polarised beam with its electric vector in the plane of the director propagate along the direction of lattice wavevector. Since thermal indexing is dependent on the angle between the electric vector of the laser and the director, it leads to nonlinear coefficient which is periodic with the period of the lattice. This leads to a periodic convergence and divergence of the incident wide beam leading to novel transformation of an otherwise parallel beam into the beam of modulated width. The modulations are schematically shown in Figure 8 (b).

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CONCLUSIONS

We have studied the optical spatial solitons in liquid crystals where the nonlinearity due to various processes exactly balances diffraction leading to solitons. We have pointed out a few of the solutions in some geometries. We suggest the existence of bright and dark soliton solutions which are similar to the Kerr soliton in case of thermal indexing alone. In the case where the laser suppression of the director fluctuations and the thermal indexing are simultaneously present we find an interesting bright kink soliton apart from the usual bell-shaped soliton. We also get solitons when the nonlinearity due to laser induced changes in the tilt angle in smectic liquid crystal and thermal indexing are both present. We next consider the critical laser power for soliton formation due to the laser suppression of the director fluctuations. *We find that the critical power is inversely proportional to the cross-section of the* beam. In the cases where thermal indexing also operates along with this process, we find solitons to occur at two critical powers.

We finally consider the effects of various nonlinearities on the width of very wide beams. We include the effect of one or more nonlinearities operating simultaneously. Here the natural diffraction of the beam is negligible. In a nematic, for a linearly polarised beam propagating perpendicular to the director with its electric vector parallel to the director, due to both the laser suppression of the director fluctuations and thermal indexing, we find periodic modulations in the beam width as it propagates in the nonlinear medium. When an unpolarised beam is incident on a homogeneously aligned sample we find that the two eigenwaves in the medium are affected differently. One of them diverges while the other is converges. In case of a hybrid aligned nematic we find an interesting transformations of the beam width. Finally, in a flexoelectric lattice with the light propagating parallel to the periodicity direction, the beam width is periodically modulated.

Acknowledgments
We wish to thank K A Suresh for useful comments.

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